

Bose-Einstein condensation in a stiff TOP trap with adjustable geometry

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We report on the realisation of a stiff magnetic trap with independently adjustable trap frequencies, ω_z and ω_r , in the axial and radial directions respectively. This has been achieved by applying an axial modulation to a Time-averaged Orbiting Potential (TOP) trap. The frequency ratio of the trap, ω_z/ω_r , can be decreased continuously from the original TOP trap value of 2.83 down to 1.6. We have transferred a Bose-Einstein condensate (BEC) into this trap and obtained very good agreement between its observed anisotropic expansion and the hydrodynamic predictions. Our method can be extended to obtain a spherical trapping potential, which has a geometry of particular theoretical interest.

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Since the first observation of BEC in a dilute alkali gas in 1995 [1], the spectroscopy of the excitations of these novel quantum objects has been an important tool for understanding them. A close analogy can be drawn between the discrete excited states of a condensate and the well defined energy levels of an atom. Spectroscopic measurements of atomic energy levels have guided the development of the quantum mechanical model of the atom and likewise the spectroscopy of the condensate should provide crucial information about its nature. The first experiments focused on low energy collective excitations both in a pure condensate close to absolute zero [2] and at finite temperature [3]. These latter experiments led to a better understanding of the interaction between the condensate and the thermal cloud [4]. The first observation of a transverse collective excitation was recently reported in [5]. This also gave evidence for superfluidity in a condensate. Recent theoretical work has provided information on the behaviour of an excited condensate in specific anisotropic trap geometries. This paper presents an adjustable magnetic trap that allows us to access these geometries whilst retaining a stiff trapping potential.

In the linear hydrodynamic theory of Bose-Einstein condensation, the spectrum of low energy collective excited states depends strongly on the trap geometry (i.e. the trap stiffnesses in all three orthogonal directions). By changing the trap frequency in one specific direction, the mode energies change relative to one another

and new features appear in the spectrum. For smaller condensates, the full Gross-Pitaevskii equation including the kinetic energy term is required. In this regime, the size of the condensate, as well as its shape, plays an important role in determining the excitation spectrum. If the shape of the potential is adjusted so that the second harmonic of one mode corresponds to the fundamental frequency of another, then the inherent non-linearity of the system leads to coupling between the modes. It has been predicted from the hydrodynamic limit of the Gross-Pitaevskii equation that this behaviour should be observed for a trap potentials with frequency ratios of 1.95, 1.51 and 0.68 [6].

Another geometry of great interest is the spherically symmetric trap, because the excitation spectrum of the condensate is simplified by degeneracy and becomes theoretically tractable [7]. One mechanism for the decay of collective excitations in a condensate is Landau damping, which is analogous to a scattering process. A condensate excitation disappears in a collision with a thermal excitation, resulting in a thermal excitation at a higher energy. The reduced availability of states in a spherical trap, caused by degeneracy, has a major effect on the Landau damping rate. Quantitative calculations of this process are currently being carried out [8].

In addition to its importance for studying the collective excitations of a BEC, the spherically symmetric harmonic trap also produces unique behaviour in a thermal cloud of atoms. The breathing mode oscillation is completely undamped in this geometry [9], whilst in an anisotropic trap all modes are strongly damped [10].

There are two general types of purely magnetic trap for neutral atoms. Ioffe-Pritchard type traps [11] use static magnetic fields to generate a very tight radial confinement. They produce prolate condensates with aspect ratios of ~ 0.05 . The aspect ratio of a condensate or thermal cloud is the ratio of the radial to axial size, and is the inverse of the trap frequency ratio, in a harmonic potential. The TOP trap [12], in its axially symmetric configuration, combines a quadrupole field with a rotating bias field to trap an oblate condensate, with an aspect ratio of 2.83. In this paper we demonstrate a novel method to independently relax the axial trapping frequency of the TOP trap. This new trap, with its adjustable aspect ratio, has been called a zTOP trap because of the configuration of bias fields that is used.

The original attempts to vary the aspect ratio of the TOP trap made use of the force of gravity on atoms held

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in a very weak magnetic potential [13]. This method, along with its practical difficulties, is discussed briefly. In contrast, the zTOP trap combines an adjustable aspect ratio with a stiff trapping potential. The experimental setup is described and finally evidence for the successful transfer of a condensate into the zTOP trap is presented.

The standard TOP trap combines a spherical quadrupole field of radial gradient B'_q , with a bias field, \mathbf{B}_t , rotating in the horizontal xy-plane. The expression for the trap potential usually ignores gravity. However, for heavy atoms such as ^{87}Rb , in a weak trap, it must be included. The full potential is given by the time-average of Eq. 1, where $r_0 = B_t/B'_q$.

$$U(x, y, z, t) = \mu B'_q \left(|(x + r_0 \cos \omega_t t) \hat{\mathbf{x}} + (y + r_0 \sin \omega_t t) \hat{\mathbf{y}} - 2z \hat{\mathbf{z}}| + \frac{mgz}{\mu B'_q} \right) \quad (1)$$

At small distances from the trap centre, $r \ll r_0$, this approximates to a harmonic trapping potential. The trap oscillation frequencies and frequency ratio are

$$f_r = \frac{1}{2\pi} \sqrt{\frac{\mu}{2m}} \frac{B'_q}{\sqrt{B_t}} (1 - \eta^2)^{1/4} (1 + \eta^2)^{1/2} \quad (2)$$

$$f_z = \frac{1}{2\pi} \sqrt{\frac{\mu}{2m}} \frac{B'_q}{\sqrt{B_t}} \sqrt{8(1 - \eta^2)^{3/4}} \quad (3)$$

$$\frac{f_z}{f_r} = \sqrt{\frac{8(1 - \eta^2)}{1 + \eta^2}} \quad (4)$$

where the parameter

$$\eta = \frac{mg}{2\mu B'_q} \quad (5)$$

is the ratio of the gravitational force on the atom to the axial magnetic force due to the quadrupole field. Note that the trap frequency ratio depends only on η . Thus by measuring the axial and radial trap frequencies, as a function of B'_q , we were able to accurately calibrate the quadrupole field strength.

However, the aspect ratio only changes significantly in very weak traps, where the gravitational force becomes important. A ten percent decrease in the aspect ratio requires $B'_q < 23 \text{ G/cm}$. Since B_t is limited by noise to a minimum value of 2 G in our apparatus, this corresponds to a radial trapping frequency of $< 15 \text{ Hz}$. Attempts were made to selectively excite quadrupole oscillations of the BEC in this weak gravity trap. However, in such a weak trap it was impossible not to excite large dipole motions, making the quadrupole oscillation impossible to observe.

The theory of the zTOP trap is analogous to that for the standard TOP trap, except that the bias field oscillates in three rather than two dimensions. Both are based on a spherical quadrupole field trap, with a trapping potential of

$$U(x, y, z) = \mu B'_q |x \hat{\mathbf{x}} + y \hat{\mathbf{y}} - 2z \hat{\mathbf{z}}| \quad (6)$$

which has a restoring force in the axial direction that is twice as strong as that in the radial direction. Now consider the effect of applying a TOP bias field rotating at ω_t in the (radial) xy-plane. This causes the locus of the quadrupole field to describe a circle in the xy-plane and after time-averaging creates an axially symmetric TOP trap. The trap stiffness is reduced in all directions compared to the quadrupole field trap, but more significantly in the radial direction, increasing the aspect ratio to 2.83. Extending this argument, the trap stiffness can be preferentially reduced in the z direction by including an oscillating axial bias field. Our present arrangement uses an axial modulation frequency (ω_a) of $2\omega_t$. Thus in our zTOP trap, the locus of the quadrupole field follows a three dimensional saddle shape rather than a circle. The total zTOP magnetic field is given in Eq. 7 as the sum of its three components - the static quadrupole field and the oscillating radial and axial bias fields.

$$\mathbf{B}(t) = B'_q (x \hat{\mathbf{x}} + y \hat{\mathbf{y}} - 2z \hat{\mathbf{z}}) + B_t (\cos \omega_t t \hat{\mathbf{x}} + \sin \omega_t t \hat{\mathbf{y}}) + B_z \cos \omega_a t \hat{\mathbf{z}} \quad (7)$$

The properties of the zTOP trap are calculated from the instantaneous zTOP potential, numerically averaged over one cycle of the bias field. This calculation gives the following constraints on the frequency, ω_a , of the axial modulation necessary to create a stable trap. Firstly, to create a time-averaged potential, ω_a must be greater than the trap frequencies. Secondly ω_a must be an exact integer multiple, n , of the radial (TOP) bias field frequency, ω_t . Our calculations showed that deviations from this condition produced asymmetry in the time-averaged potential between the x and y directions. In the xy-plane, the trap will acquire a slightly elliptical, rotating cross-section, whilst its centre will describe a small circle. These slow variations in the time-averaged potential at the beat frequency, $\omega_a - n\omega_t$, could heat the cloud. It is also possible to choose ω_t as an integer multiple of ω_a . However this configuration requires the two signals to be phase locked, if the 'micromotion' of the trap described above is to be avoided. In addition, ω_a must be lower than the Larmor frequency, so that Majorana spin flips are not induced.

The extent of the trap relaxation in the axial direction depends on the waveform and amplitude, B_z , of the axial modulation, but not on its frequency. The axial modulation causes the locus of the quadrupole field to oscillate in the vertical direction. A waveform that causes it to spend more time per cycle at the extremes of this motion results in a more relaxed trap in the axial direction. Thus an ideal square wave produces the weakest trap, for a given maximum B_z field. Removing the higher harmonics in the axial modulation stiffens the trap as shown in Fig.1. Note that significant additional relaxation, with respect to single frequency modulation, is achieved by adding only the third harmonic component of a square wave to the fundamental. In this case a spherical trap is produced for $B_z/B_t = 5.5$. Since B_t is typically 2 G,

this is a readily achievable experimental condition. The axial trapping frequency is also reduced by increasing the modulation amplitude, B_z . Thus we are able to vary the aspect ratio of the trap during the course of an experiment by controlling the peak voltage across the axial bias coils.

The apparatus that we use to create ^{87}Rb condensates is described in detail elsewhere [14]. In summary, we use a differentially pumped double MOT system (which incorporates a pyramidal MOT [15]) to load a TOP trap with $\sim 2 \times 10^8$ atoms. Evaporative cooling proceeds via both Majorana spin flip and radio frequency cutting to achieve condensates with $\sim 2 \times 10^4$ atoms.

To generate the axial bias field, two Helmholtz coils of 35 turns each, have been added above and below the experimental cell. A 2 turn pickup coil is also present to monitor the applied field. We currently use an axial modulation at 14 kHz ($2\omega_t$). The amplitude of this signal is computer controlled and fed to an audio amplifier. A transformer is used to match the output of the amplifier to the coils. Fine adjustments are made to the frequency until the x and z pickup coils produce a clean Lissajous figure on the oscilloscope, indicating that the axial oscillation is at exactly twice the frequency of the radial bias field. This frequency relationship is stable to within ~ 1 Hz over the day.

The trap frequencies were calibrated with respect to the voltage supplied across the axial bias field coils. A small, cold thermal cloud was collected in a 'displaced' zTOP trap. The additional static magnetic fields causing the displacement were suddenly switched off, exciting orthogonal dipole oscillations at the two trap frequencies. The radial and axial positions of the cloud were recorded as a function of time, using absorption imaging, and fitted with sine waves to obtain the trap frequencies. Figure 2 shows the data for the normalised trap frequencies and the trap frequency ratio, for a range of axial bias field amplitudes.

Theoretical values for the normalised trap frequencies, as a function of B_z/B_t , were found by numerically integrating the magnetic potential energy of an atom over one cycle of the bias field. These values were accurately described by an eighth order polynomial. Two free fitting parameters were used to fit this polynomial to the experimental data. One parameter determined the trap frequencies without any axial bias field and the other was a constant, relating the voltage across the axial bias coils to the actual value of B_z/B_t that was produced. Fitting this expression to the axial trap frequency data gave values for both fitting parameters and their respective errors. The solid lines in Fig.2 show the fitted zTOP trap calibration curves.

The condensate is initially produced by a sequence of computer controlled evaporative cooling ramps in a standard TOP trap, with final trap frequencies $\omega_r = 126$ Hz and $\omega_z = 356$ Hz ($= \sqrt{8}\omega_r$). The amplitude of the oscillating axial field is then linearly ramped from zero to its final value over 0.5s. This adiabatically transfers the

condensate into the zTOP trap.

Figure 3 shows the aspect ratio of a BEC, 12ms after release from zTOP traps with a range of frequency ratios. Changing the frequency ratio of the trap changes the distribution of the self energy of the condensate among the three orthogonal directions and hence the form of the free expansion of the condensate. The aspect ratio of the expanding BEC has been predicted as a function of trap geometry using the hydrodynamic equations [16] and is shown as a solid line in Fig. 3. This theoretical curve contains no free parameters. The good agreement between theory and experiment confirms two facts. Firstly that the condensed state survived adiabatic transfer to the zTOP trap and secondly that the trap potential experienced by the condensate has been modified as predicted.

In conclusion, we have modified our TOP trap to create a stiff zTOP trap with an adjustable geometry. Trap frequency ratios between 2.83 and 1.6 have been demonstrated by applying a magnetic field in the axial direction, oscillating at a single frequency. The addition of higher harmonics to the axial bias field modulation will make a spherical trap accessible. This prediction, combined with the successful transfer of BECs into the zTOP trap, as demonstrated in this paper, promises to open up the field of degenerate condensate excitations to experimental research.

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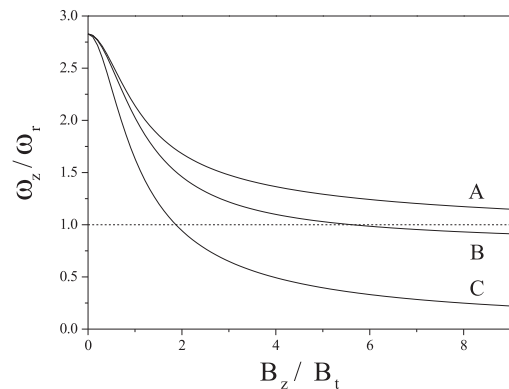


FIG. 1. Simulations of the trap frequency ratio ω_z/ω_r as a function of the B_z/B_t . The solid lines correspond to different B_z waveforms, (A) fundamental only, (B) fundamental + third harmonic component of a square wave, (C) square wave. The dotted line indicates the value of B_z/B_t required for a spherically symmetric trap in each case.

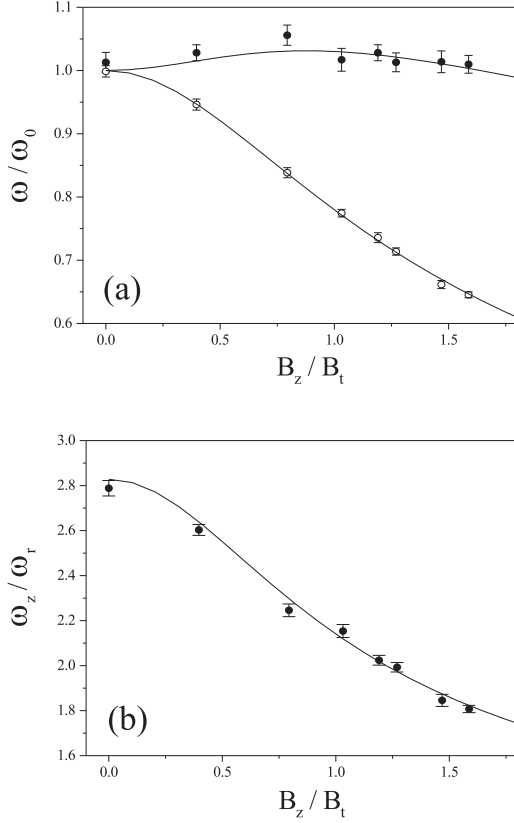


FIG. 2. Experimental values for the normalised trap frequencies and aspect ratio as a function of B_z/B_t , with the theoretical curve that was used to calibrate the zTOP trap. (a) shows how the normalised individual trap frequencies, ω_r/ω_{r0} (solid circles) and ω_z/ω_{z0} (open circles), depend on B_z/B_t . ω_{r0} and ω_{z0} are the radial and axial trap frequencies when $B_z = 0$. (b) shows the aspect ratio of the trap as a function of B_z/B_t . For all points, $B_t = 2G$.

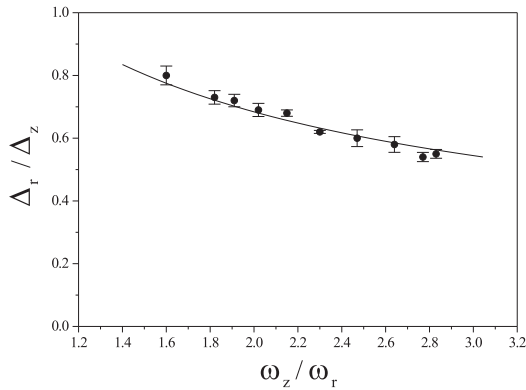


FIG. 3. The aspect ratio of a BEC, $(\Delta r/\Delta z)$, after 12ms time-of-flight, as a function of the zTOP trap frequency ratio (ω_z/ω_r) . The solid line shows the solution of the hydrodynamic equations as a function of trap geometry (no free parameters). Each data point is the mean of six shots and the error bar shows the standard deviation of the mean.

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